MATH-803 Geometry

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: After having completed this course, the students would be expected to understand classical concepts in the local theory of curves, surfaces and manifolds. Also the students will be familiar with the geometrical interpretation of the terminology used in the course.

Detailed Course Contents: Curves, Surfaces -Topological Invariants, Geometry on a Surface or Riemannian Geometry, Geodesics, Generalization of the Concept of Tangent and of Tangent Plane, to a Surface Manifolds -Tensor Fields - Covariant Differentiation, Tangent Vectors and Mappings, Tangent or Contravariant" Vectors, Vectors as Differential Operators, The Tangent Space to Mn at a Point, Change of Coordinates, Vector Fields and Flows on Rn, Vector Fields on Manifolds, Functionals and the Dual Space, The Differential of a Function, Scalar Products in Linear Algebra, Riemannian Manifolds and the Gradient Vector, The Tangent Bundle, The Cotangent Bundle and Phase Space, Covariant Tensors, Contravariant Tensors, Mixed Tensor, Properties, The Tensor Product of Covariant Tensors, Wedge Product, The Geometric Meaning, Special Cases, Computations and Vector Analysis, The Exterior Differential, A Coordinate Expression ford, The Pull-Back of a Covariant Tensor,. Integration of a p-Form in Rp, Integration with boundaries, Stokes' theorem, The Lie Bracket, The Lie Derivatives of Forms
Covariant Derivative, Curvature of an Affine Connection, Geodesics.

Course Outcomes: Students are expected to understand classical concepts in the local theory of curves, surfaces and manifolds. Also the students will be familiar with the geometrical interpretation of the terminology used in the course. Students will be able to apply learned concepts in other related fields.

Textbooks:
1. T. Frankel, the Geometry of Physics, Cambridge University Press, 2012 (TB2).
2. A. Visconti, Introductory Differential Geometry for Physicists, World Scientific, 1992 (TB1).

Reference Books:
1. Bernard F. Schutz , Geometrical Methods of Mathematical Physics, Cambridge University Press, 1980.
2. Serge Lang, Fundamentals of Differential Geometry, Springer, 1999.

Weekly Breakdown		
Week	***Section***	***Topics***
1	1(TB2)	Curves, handouts
2	2(TB2)	Surfaces -Topological Invariants,
3	3(TB2)	Geometry on a Surface or Riemannian Geometry
4	4(TB2)	Geodesics
5	5(TB2)	Generalization of the Concept of Tangent and of Tangent Plane to a Surface
6	**First One Hour Test**	
7	(TB1,TB2) 1.1a,1.2(a-c) 1.3(a-c)	Manifolds -Tensor Fields - Covariant Differentiation Tangent Vectors and Mappings, Tangent or “Contravariant” Vectors, Vectors as Differential Operators, The Tangent Space to Mn at a Point
8	(TB1)1.4(a-b)	Change of Coordinates, Vector Fields and Flows on Rn, Vector Fields on Manifolds
9	(TB1)2.1(a-d)	Functionals and the Dual Space, The Differential of a Function, Scalar Products in Linear Algebra, Riemannian Manifolds and the Gradient Vector
10	(TB1)2.2a, 2.3(a-b)	The Tangent Bundle, The Cotangent Bundle and Phase Space
11	(TB1)2.4(a-e).	Covariant Tensors, Contravariant Tensors, Mixed Tensor, Properties
12	**Second One Hour Test**	
13	(TB1)2.5(a-e) 2.6(a-c)	The Tensor Product of Covariant Tensors, Wedge Product, The Geometric Meaning, Special Cases, Computations and Vector Analysis. The Exterior Differential, A Coordinate Expression for D,
14	(TB1)2.7a, 3.1a, 3.2, 3.3	The Pull-Back of a Covariant Tensor,. Integration of a p-Form in Rp, Integration with boundaries, Stokes’ theorem
15	(TB1)4.1, 4.2a.	The Lie Bracket, The Lie Derivatives of Forms
16	(TB1)9.1(a-c)	Covariant Derivative, Curvature of an Affine Connection Godesics
17		Review
18	**End Semester Exam**	

MATH-804 Differential Equations

Credit Hours: 3-0
Prerequisite: None

Course Objectives: Differential equations (DE) are the language in which the laws of nature are expressed and these now have applications in biology, medicine, sociology, psychology, economics, chemistry, physics and engineering. Methods for solving DEs are therefore of fundamental importance for understanding nature and technology.
In this course, the objective is to learn how to deal with second-order ordinary differential equations with variable coefficients, understand the properties of solutions and also, how to find solutions that obey prescribed boundary conditions. Not all DEs can be solved in terms of known functions such as polynomials, exponentials and the like. A major aim of this course is to teach the student how to get information about the solution in these cases using power series methods and Frobenius method. A second major aim is to learn how to find solutions to boundary value problems using Sturm-Liouville methods.

Core Contents: Series solution method, Bessel functions, Eigenvalue problems, Phase plane and stability.

Detailed Course Contents: Series solution method: Introduction, power series, Taylor series, ordinary points, regular singular points, convergence
Bessel functions: the Gamma function, Bessel's equation, Bessel function of the first, second and third kind, properties of Bessel functions, modified Bessel functions
Orthogonal polynomials: Sets of orthogonal polynomials and their properties, generating functions, Legendre, Hermite, Laguerre and Chebyshev polynomials and their properties, differential equations satisfied by these polynomials
Eigenvalue problems: Self-adjoint problems, singular problems
Phase plane and stability

Course Outcomes: At the end of the course, the student should be:
- Able to recognize various types of differential equations.
- Able to choose the appropriate method to solve the given differential equation.
- Able to deal with higher-order differential equations.
- Able to analyze the behavior of the solution.
- Able to solve the systems and show the behavior of the solution in phase-plane.
- Familiar with the properties of polynomials and sets of polynomials which are important in applied sciences.

Textbooks:

1. Albert L. Rabenstein, Introduction to Ordinary Differential Equations, Second EnlargedEdition with Applications Academic Press Inc., 1972.
2. RHB: K. F. Riley, M. P. Hobson and S. J. Bence, Mathematical Methods of Physics andEngineering, 3rd edition, 2006, (Chapter 22)

Reference Books: Denis G. Zill, Differential Equations with Boundary-Value Problems, 9th edition, Cengage Publishing, 2018

Weekly Breakdown		
Week	***Section***	***Topics***
1	ALR 3.1 – 3.3	Power Series Solutions of linear differential equations about the ordinary points.
2	ALR 3.4– 3.8	Solutions about regular singular points, indicial equation, cases of distinct and equal roots, point at infinity
3	ALR 4.1-4.2	The Gamma Function, Series solutions of the Bessel’s equation
4	ALR 4.3, 4.4	Properties of Bessel functions, Bessel functions of the second and third kinds
5	ALR 4.5, 4.6	Modified Bessel’s function, other forms of Bessel functions
6	**First One Hour Test**	
7	ALR 5.1-5.3	Orthogonal functions, an existence theorem for orthogonal polynomials, properties of orthogonal polynomials
8	ALR 5.4, 5.5	Generating functions, the generating function for the Legendre polynomials
9	ALR 5.6, 5.7	Properties of the Legendre polynomials Orthogonally of the Legendre polynomials
10	ALR 5.8-5.10	The Legendre differential equation, Hermite, Laguerre and Chebyshev polynomials
11	ALR 6.1	Eigenvalue problems for differential equations
12	**Second One Hour**	
13	ALR 6.2-6.3	Self adjoint problems (Sturm-Liouville Problem), Some special Cases
14	ALR 6.4 – 6.5	Singular Problems, Some important singular problems
15	RHB 22.6	Variational formulation of an SL problem, Rayleigh-Ritz method for the estimation of the lowest eigenvalues.
16	ALR 11.1 – 11.2	Definition of stability, stability of linear systems
17	ALR 11.3	Positive definite and negative definite functions, Method of Lyapunov
18	**End Semester Exam**	

MATH-822 Mathematical Modelling-I

Credit Hours: 3-0
Prerequisite: None

Course Objectives: The course focuses on the application of "dimensional methods" to facilitate the design and testing of engineering problems. It aims to develop a practical approach to modeling and dimensional analysis. This course will be well received and will prove to be an invaluable reference to researchers and students with an interest dimensional analysis and modeling and those who are engaged in design, testing and performances evaluation of engineering and physical system.

Core Contents: The course includes the theory of matrix algebra and linear algebra, the theory of dimension, transformation of dimensions and structure of physical variables, dimensional similarities and models law. This course will cover the nature of dimensional analysis use in mathematical modeling.

Detailed Course Contents: Mathematical Preliminaries, Matrices and Determinants, Operation with Matrices, The rank of matrices and Systems of linear equations, Formats and Classification, Numerical, Symbolic and Mixed format, Classification of Physical Quantities, dimensional system, General Statement, The SI system, Structure, Fundamental dimension, Derived dimensional units with and without specific names, Rules of etiquettes in Writing dimensions.
Other than SI dimensional systems, A note on the classification of dimensional systems, Transformation of Dimensions, Numerical equivalences, Techniques, Examples, Problems, Arithmetic of Dimensions, and Dimensional Homogeneity.
Equations, graphs, Problems, Structure of Physical Relations, the dimensional matrix, Number of independent sets of products of given dimension 1,11, Special case, Buckingham's theorem, Selectable and non-selectable dimensions, Minimum number of independent product of variables of given dimension, Constancy of the sole dimensionless product, Number of dimension equals or exceeds the number of variables, Systematic determination of Complete Set of Products of Variable Transformations, Theorems related to some specific transformations, Transformations between systems of different d matrices, Number of Sets of Dimensionless Products of Variables
Distinct and equivalent sets, Changes in dimensional set not affecting the dimensional variables, prohibited changes in dimensional set.
Relevancy of Variables, Dimensional irrelevancy, Condition, Adding a dimensionally irrelevant variables to a set of relevant variables, Physical irrelevancy, Problems, Economy of Graphical Presentation, Number of curves and charts, Problems, Forms of Dimensionless Relations
General classification, Monomial is Mandatory, Monomial is impossible, Reconstructions, Sequence of Variables in the Dimensional Set, Dimensionless physical variable is present, Physical variables of identical dimensions are present, Independent and dependent variables.

Learning Outcomes: Students are expected to understand fundamentals dimension of dimensional analysis.

Textbooks:

1. Thomas Szitres, Applied Dimensional Analysis and Modeling, Elsevier Inc., 2007.(Referred as TS).
2. S.H. Friedberg, A.J. Insel, L.E.Spence, Linear Algebra, Prentice-Hall, Inc., EnglewoodCliffs, N.J. USA,979 (referred as FIS)

Weekly Breakdown		
Week	***Section***	***Topics***
1	TS, Ch. 1, FIS, Ch. 3	Mathematical Preliminaries, Matrices and Determinants, Operation with Matrices, The rank of matrices and Systems of linear equations
2	TS Chs. 2, 3	Formats and Classification, Numerical, Symbolic and Mixed format Classification of Physical Quantities, dimensional system, General Statement, The SI system
3	Ch 3	Structure, Fundamental dimension, Derived dimensional units with and without specific names, Rules of etiquettes in Writing dimensions Other than SI dimensional systems
4	Chs 3,4	A note on the classification of dimensional systems, Transformation of Dimensions, Numerical equivalences, Techniques, Examples, Problems
5	Chs 5, 6	Arithmetic of Dimensions, Dimensional Homogeneity
6	**First One Hour Test**	
7	Chs 6,, 7	Equations, graphs, Problems, Structure of Physical Relations, the dimensional matrix, Number of independent sets of products of given dimension 1,11, Special case
8	Ch 7	Buckingham's theorem, Selectable and non-selectable dimensions, Minimum number of independent product of variables of given dimension, Constancy of the sole dimensionless product
9	Chs 7,8	Number of dimension equals or exceeds the number of variables Systematic determination of Complete Set of Products of Variable
10	Ch 9	Transformations, Theorems related to some specific transformations, Transformations between systems of different d matrices
11	Ch 10	Number of Sets of Dimensionless Products of Variables Distinct and equivalent sets, Changes in dimensional set not affecting the dimensional variables, Prohibited changes in dimensional set
12	**Second One Hour Test**	
13	Ch 11	Relevancy of Variables, Dimensional irrelevancy, Condition, Adding a dimensionally irrelevant variables to a set of relevant variables,
14	Chs 11, 12	Physical irrelevancy, Problems, Economy of Graphical Presentation Number of curves and charts, Problems
15	Ch 13	Forms of Dimensionless Relations, General classification, Monomial is Mandatory, Monomial is impossible, Reconstructions
16	Ch 14	Sequence of Variables in the Dimensional Set Dimensionless physical variable is present, Physical variables of identical dimensions are present, Independent and dependent variables
17		Review of Material
18	**End Semester Exam**	

MATH-824 Applied Optimization Methods

Credit Hours: 3-0
Prerequisite: None

Objectives and Goals: The purpose of optimization is to achieve the "best" design relative to

A set of prioritized criteria or constraints. The proposed course introduces students to analyze solutions to complex applied problems and acquire the best solution. Explain the fundamental knowledge of Linear Programming and Dynamic Programming problems. This course enables to identify, formulate, research through relevant literature review, and solve engineering problems reaching substantiated conclusions. It includes the basics of different evolutionary algorithms.

Core Contents: Contents cover linear programming, the simplex method, fundamentals of unconstrained optimization techniques, and different evolutionary algorithms.

Detailed Course Contents: Basic definitions and introduction to optimization, graphical optimization, linear and non-linear optimization, iterative algorithms, Simplex method and duality problems, constrained nonlinear optimization, Lagrange multipliers, and linear inequality constraints kkt condition, non-traditional optimization algorithms, genetic algorithms.

Course Outcomes: Upon successful completion of the course, the student will demonstrate competency by being able to:

1. Explain the fundamental knowledge of Linear Programming and Dynamic Programming problems.
2. Apply the theory of optimization methods and algorithms to develop and for solving various types of optimization problems.
3. Go into research by applying optimization techniques in problems of Engineering and Technology.
4. Identify, formulate, research through relevant literature review, and solve engineering problems reaching substantiated conclusions.
5. Describe the basics of different evolutionary algorithms.

Textbooks:

1. S. S. Rao. "Engineering Optimization: Theory and Practice." John Wiley & Sons, Inc., 5th Edition

Reference Books:

1. E. K. P. Chong, H. S. Żak, "An Introduction to Optimization". John Wiley & Sons Latest Edition
2. J. S. Arora, "Introduction to Optimum Design," McGraw-Hill, New York, 1989

Weekly Breakdown		
Week	***Section***	***Topics***
1	1.1-1.3	Convexity, Derivatives, Gradient, Hessian, Jacobian, Optimization overview,
2	1.4-1.7	Optimization problem formulation (Design Variables, Constraints, Objective Function, Variable Bounds)
3	2.1-2.5	Linear and non-linear optimization, Definitions of global and local minima, Taylor's Expansion,
4	3.1-3.5	Quadratic forms, and definite Matrices, the concept of necessary and sufficient conditions
5	5.12	Newton's method for a system of equations, Taylor's theorem in N-dimensions,
6	**First One Hour Test**	
7	6.1	Newton convergence fractals, and rate of convergence.
8	3.6-3.8	Linear constraints, linear programming problems,
9	3.9-3.10	Standard form, basic feasible solutions, Simplex method,
10	4.2-4.3	Dual problem, strong duality, unconstrained optimization.
11	6.1-6.2	Non-linear optimization, linear equality constraints, optimality, reduced Newton
12	**Second One Hour Test**	
13	6.8-6.10	Lagrange multipliers, and linear inequality constraints
14	2.5-2.6, 10.1-10.3	Karush–Kuhn–Tucker (KKT) conditions, GA operators and Population selection,
15	12.1-12.4	Fitness function, Crossover, mutation, etc. Stopping criteria,
16	13.1-13.2	Some applications of optimization techniques
17	13.3-13.6	Some applications of optimization techniques
18	**End Semester Exam**	

PHY-907 General Relativity

Credit Hours: 3-0
Prerequisite: None

Course Objectives: General Relativity (GR) is a physical theory of gravitation invented by Albert Einstein in the early twentieth century. The theory has strong mathematical setup, has immense predictive power, and has successfully qualified several experimental/observational experiments of astrophysics and cosmology. Black holes and relativistic cosmology are two main applications of GR. It is intended that GR and its major applications and achievements be discussed in the manner they deserve.

Core Contents: Special relativity revisited, Electromagnetism, The gravitational field equations, The Schwarzschild geometry, Schwarzschild black holes, Kerr metric, Further spherically symmetric geometries.

Detailed Course Contents: Special relativity revisited: Minkowski spacetime in Cartesian coordinates, Lorentz transformations, Cartesian basis vectors, Four-vectors and the lightcone, Four-vectors and Lorentz transformations, Four-velocity, Four-momentum of a massive particle, Four-momentum of a photon, The Doppler effect and relativistic aberration, Relativistic mechanics, Free particles, Relativistic collisions and Compton scattering, Accelerating observers, Minkowski spacetime in arbitrary coordinates.
Electromagnetism: The electromagnetic force on a moving charge, The 4-current density, The electromagnetic field equations, Electromagnetism in the Lorenz gauge, Electric and magnetic fields in inertial frames, Electromagnetism in arbitrary coordinates, Equation of motion for a charged particle, Electromagnetism in a curved spacetime.
The gravitational field equations: The energy–momentum tensor, The energy–momentum tensor of a perfect fluid, Conservation of energy and momentum for a perfect fluid, The Einstein equations, The Einstein equations in empty space, The weak-field limit of the Einstein equations, The cosmological-constant term.
The Schwarzschild geometry: General static isotropic metric, Schwarzschild solution, Birkhoff's theorm, Gravitational redshift, geodesics in Schwarzschild geometry, radial trajectories of massive particles, Circular motion of massive particles, stability of massive particle orbits, trajectories of photons, circular motion of photons, stability of photon orbits, Experimental tests of general relativity: Precession of planetary orbits, The bending of light, Accretion discs around compact objects.
Schwarzschild black holes: singularities in Schwarzschild metric, radial photon worldlines, radial particle worldliness in Schwarzschild coordinates, Eddington Finkelstein coordinates, black hole formation, Spherically symmetric collapse of dust, tidal forces near a black hole, Kruskal coordinates, wormholes and Einstein Rosen bridge, The Hawking effect of black hole evaporation.

Further spherically symmetric geometries: Spherically symmetric geometries: metric for stellar interior, relativistic equations of stellar structure, Schwarzschild interior solution, metric outside a spherically symmetric charged mass, Riessner-Nordstrom geometry and solution, Radial photon trajectories in RN geometry, radial massive particle trajectories.
Kerr metric: The Kerr metric, Limits of the Kerr metric, Ker Neumann Metric (handouts). The Friedmann–Robertson–Walker geometry: The cosmological principle, synchronous commoving coordinates, homogeneity and isotropy of the universe, maximally symmetric 3-space, Friedmann-Robertson-Walker metric, geometrical properties of FRW metric, The cosmological redshift, The Hubble and deceleration parameters, Components of the cosmological fluid, Cosmological parameters, The cosmological field equations, General dynamical behaviour of the universe, Evolution of the scale factor, Analytical cosmological models.

Learning Outcomes: Students will understand of the theory and predictions of Einstein's general relativity. Students will be capable to read research papers and initiate research in general relativity. Students will be able to understand the dynamical evolution of the universe by studying cosmology.

Textbook: M.P. Hobson, G.P. Efstathiou, A.N. Lasenby, General Relativity, Cambridge University Press (2007). Refereed as: HEL

Reference Books:

Weekly Breakdown		
Week	***Section***	***Topics***
1	HEL 5.1-5.7	Special relativity revisited: Minkowski spacetime in Cartesian coordinates, Lorentz transformations, Cartesian basis vectors, Four-vectors and the lightcone, Four-vectors and Lorentz transformations, Four-velocity, Four-momentum of a massive particle.
2	HEL 5.8-5.14	Four-momentum of a photon, The Doppler effect and relativistic aberration, Relativistic mechanics, Free particles, Relativistic collisions and Compton scattering, Accelerating observers, Minkowski spacetime in arbitrary coordinates.
3	HEL 6.1-6.4	Electromagnetism: The electromagnetic force on a moving charge, The 4-current density, The electromagnetic field equations, Electromagnetism in the Lorenz gauge.
4	HEL 6.5-6.7	Electric and magnetic fields in inertial frames, Electromagnetism in arbitrary coordinates, Equation of motion for a charged particle, Electromagnetism in a curved spacetime.
5	HEL 8.1-8.7	The gravitational field equations: The energy–momentum tensor, The energy–momentum tensor of a perfect fluid, Conservation of energy and momentum for a perfect fluid, The Einstein equations, The Einstein equations in empty space, The weak-field limit of the Einstein equations, The cosmological-constant term.
6	First One Hour Test	
7	HEL 9.1-9.7	The Schwarzschild geometry: General static isotropic metric, Schwarzschild solution, Birkhoff's theorm, Gravitational redshift, geodesics in Schwarzschild geometry, radial trajectories of massive particles.

8	HEL 9.8-9.13	Circular motion of massive particles, stability of massive particle orbits, trajectories of photons, circular motion of photons, stability of photon orbits.
9	HEL 10.1, 10.2, 10.4	Experimental tests of general relativity: Precession of planetary orbits, The bending of light, Accretion discs around compact objects.
10	HEL 11.1 - 11.6	Schwarzschild black holes: singularities in Schwarzschild metric, radial photon worldlines, radial particle worldliness in Schwarzschild coordinates, Eddington Finkelstein coordinates, black hole formation.
11	HEL 11.7 - 11.11	Spherically symmetric collapse of dust, tidal forces near a black hole, Kruskal coordinates, wormholes and Einstein Rosen bridge, The Hawking effect of black hole evaporation.
13	HEL 12.1-12.6	Further spherically symmetric geometries: Spherically symmetric geometries: metric for stellar interior, relativistic equations of stellar structure, Schwarzschild interior solution, metric outside a spherically symmetric charged mass, Riessner-Nordstrom geometry and solution
14	HEL 12.7-12.8 13.5, 13.6	Radial photon trajectories in RN geometry, radial massive particle trajectories, Kerr metric: The Kerr metric, Limits of the Kerr metric, Ker Neumann Metric (handouts).
15	HEL 14.1-14.7	The Friedmann–Robertson–Walker geometry: The cosmological principle, synchronous commoving coordinates, homogeneity and isotropy of the universe, maximally symmetric 3-space, Friedmann-Robertson-Walker metric, geometrical properties of FRW metric.
16	HEL 14.9, 14.10	The cosmological redshift, The Hubble and deceleration parameters.
17	HEL 15.1-15.6	Components of the cosmological fluid, Cosmological parameters, The cosmological field equations, General dynamical behaviour of the universe, Evolution of the scale factor, Analytical cosmological models.